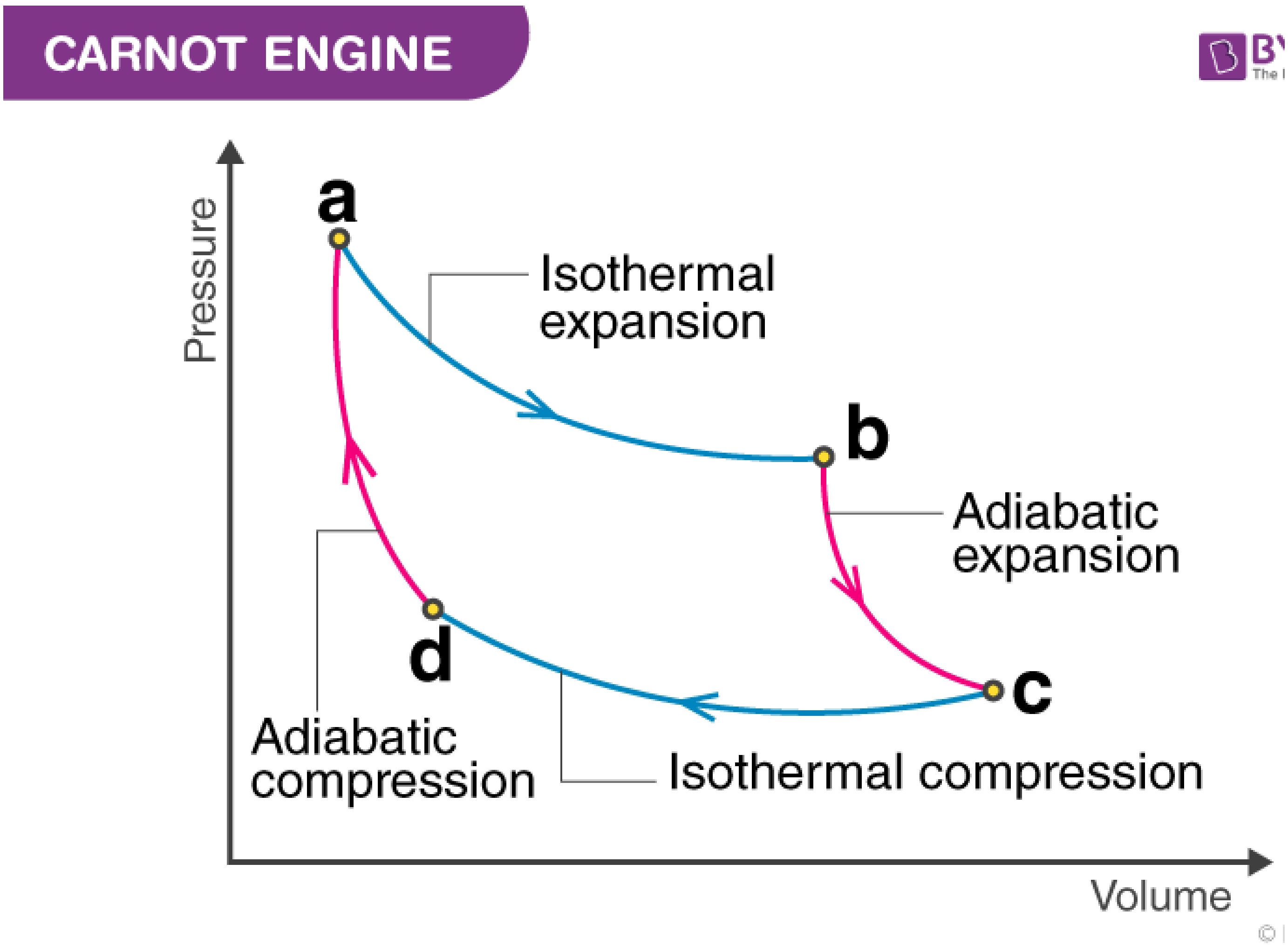
Carnot Engine - Thermodynamic Engine

What is a Carnot Engine?

Carnot engine is a theoretical thermodynamic cycle proposed by **Leonard Carnot**. It gives the estimate of the maximum possible efficiency that a heat engine during the conversion process of heat into work and conversely, working between two reservoirs, can possess. In this section, we will learn about the Carnot cycle and Carnot Theorem in detail.



Carnot Theorem:

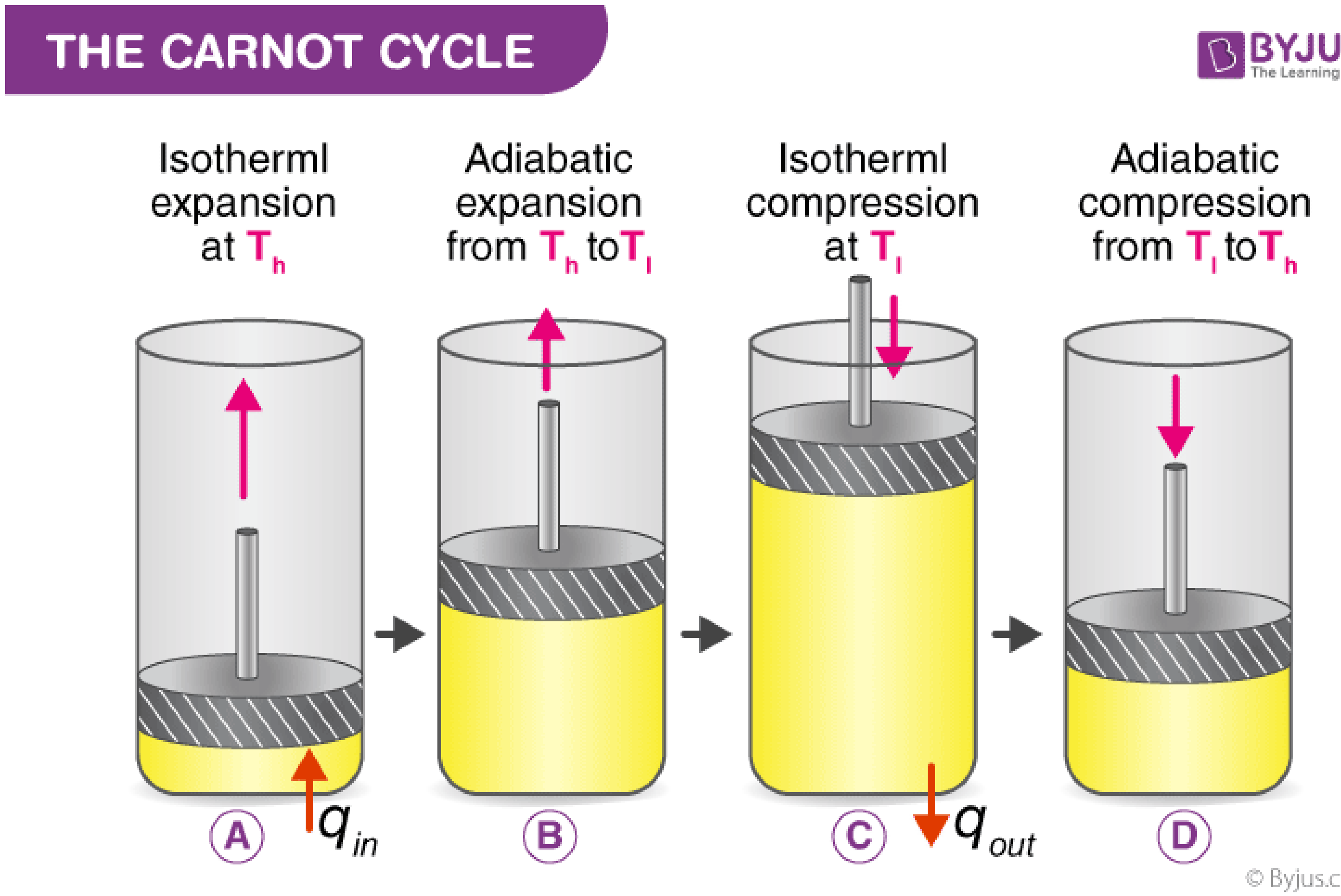
According to Carnot Theorem:

> Any system working between two given temperatures T_1 (hot reservoir) and T_2 (cold reservoir), can never have an efficiency more than the Carnot engine working between the same reservoirs respectively.

Also, the efficiency of this type of engine is independent of the nature of the working substance and is only dependent on the temperature of the hot and cold reservoirs.

Carnot Cycle:

A Carnot cycle is defined as an ideal reversible closed thermodynamic cycle in which there are four successive operations involved and they are isothermal expansion, adiabatic expansion, isothermal compression, and adiabatic compression. During these operations, the expansion and compression of substance can be done up to desired point and back to initial state.



Following are the four processes of Carnot cycle:

- In (a), the process is reversible isothermal gas expansion. In this process, the amount of heat absorbed by the ideal gas is q_{in} from the heat source which is at a temperature of T_h. The gas expands and does work on the surroundings.
- In (b), the process is reversible adiabatic gas expansion. Here, the system is thermally insulated and the gas continues to expand and work is done on the surroundings. Now the temperature is lower, T_l.
- In (c), the process is reversible isothermal gas compression process. Here, the heat loss, q_{out}occurs when the surroundings do the work at temperature T_l.
- In (d), the process is reversible adiabatic gas compression. Again the system is thermally insulated. The temperature again rise back to T_h as the surrounding continue to do their work on the gas.

Steps involved in a Carnot Cycle

For an ideal gas operating inside a Carnot cycle, the following are the steps involved:

Step 1:

Isothermal expansion: The gas is taken from P_1, V_1, T_1 to P_2, V_2, T_2. Heat Q_1 is absorbed from the reservoir at temperature T_1. Since the expansion is isothermal, the total change in internal energy is zero and the heat absorbed by the gas is equal to the work done by the gas on the environment, which is given as:

W1→2=Q1=μ×R×T1×lnv2v1

Step 2:

Adiabatic expansion: The gas expands adiabatically from P_2, V_2, T_1 to P_3, V_3, T_2.

Here work done by the gas is given by:

W2→3=μRγ−1(T1−T2)

Step 3:

Isothermal compression: The gas is compressed isothermally from the state (P_3, V_3, T_2) to (P_4, V_4, T_2).

Here, the work done on the gas by the environment is given by:

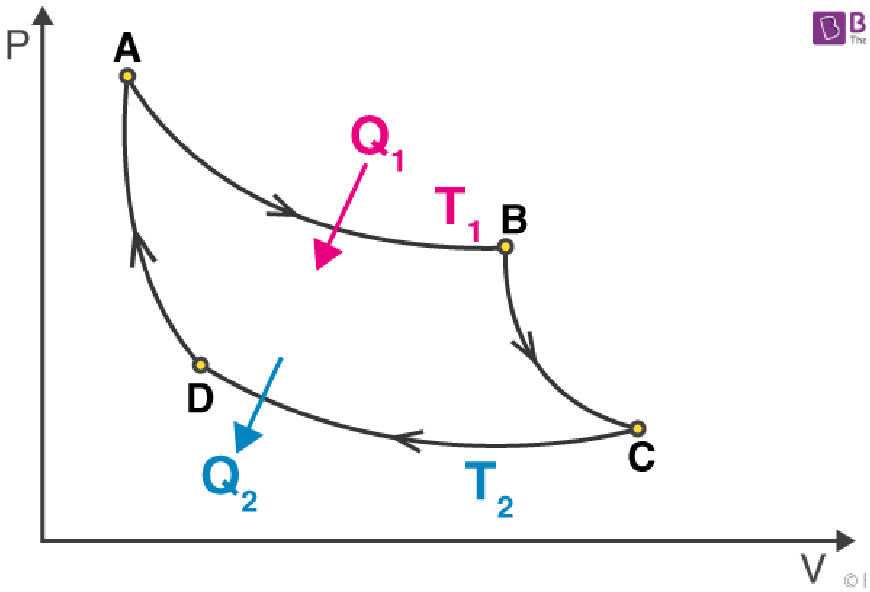
W3→4=μRT2lnv3v4

Step 4:

Adiabatic compression: The gas is compressed adiabatically from the state (P_4, V_4, T_2) to (P_1, V_1, T_1).

Here, the work done on the gas by the environment is given by:

W4→1=μRγ−1(T1−T2)



Hence, the total work done by the gas on the environment in one complete cycle is given by:

$W = W_{1\to2} + W_{2\to3} + W_{3\to4} + W_{4\to1}$ $W = \mu RT_1 \ln\frac{v_2}{v_1} - \mu RT_2 \ln\frac{v_3}{v_4}$

$\text{Net efficiency} = \frac{\text{Net work done by the gas}}{\text{Heat absorbed by the gas}}$

$\text{Net efficiency} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}\frac{\ln\frac{v_3}{v_4}}{\ln\frac{v_2}{v_1}}$

Since the step 2–>3 is an adiabatic process, we can write $T_1V_2^{Y-1} = T_2V_3^{Y-1}$

Or,

$\frac{v_2}{v_3} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$

Similarly, for the process 4–>1, we can write

$\frac{v_1}{v_2} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$

This implies,

$\frac{v_2}{v_3} = \frac{v_1}{v_2}$

So, the expression for net efficiency of carnot engine reduces to:

$\text{Net efficiency} = 1 - \frac{T_2}{T_1}$